

Solutions to Maths workbook - 1 | Quadratic Equations

MEQB

Level - 1

- 1.(C)** $p = 13, q = (-2)(-15) = 30; \quad x^2 + px + q = 0 \Rightarrow x^2 + 13x + 30 = 0 \Rightarrow x = -3, -10$
- 2.(C)** $(x-a+b)^2 + (x-b+c)^2 = 0 \Rightarrow (x-a+b)^2 = 0 = (x-b+c)^2 \Rightarrow x-a+b = 0 = x-b+c$
 $\Rightarrow x = a-b, x = b-c \Rightarrow a-b = b-c \Rightarrow a, b, c$ are in A.P.
- 3.(B)** $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r} \Rightarrow (x+p)(x+q) = r(2x+p+q) \Rightarrow x^2 + (p+q-2r)x + pq - qr - pr = 0$
 Sum of the roots $= 0 \Rightarrow p+q-2r = 0 \Rightarrow p+q+r = 3r$
- 4.(A)** The roots of $x^2 - 2mx + m^2 - 1 = 0$ are $\frac{2m \pm \sqrt{4m^2 - 4(m^2 - 1)}}{2} = m \pm 1$
 Given $-2 < m-1, m+1 < 4 \Rightarrow m > -1, m < 3 \Rightarrow -1 < m < 3$
- 5.(A)** Discriminant $= (c-a)^2 - 4(b-c)(a-b) = c^2 + a^2 - 2ca - 4ab + 4b^2 + 4ac - 4bc$
 $= a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac = (a-2b+c)^2 \geq 0 \quad \therefore$ The roots are real.
- 6.(D)** $y = \frac{x}{x^2 - 5x + 9} \Rightarrow yx^2 - (5y+1)x + 9y = 0 \Rightarrow (5y+1)^2 - 4y(9y) \geq 0 \Rightarrow -11y^2 + 10y + 1 \geq 0$
 $\Rightarrow 11y^2 - 10y - 1 \leq 0 \Rightarrow (11y+1)(y-1) \leq 0 \Rightarrow -\frac{1}{11} \leq y \leq 1 \quad \therefore$ Maximum value $= 1$
- 7.(B)** If α, β are the roots then $\alpha + \beta = a-2, \alpha\beta = -a-1$
 Sum of the squares of the roots, $S = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a-2)^2 - 2(-a-1)$
 S is least $\Rightarrow$ when $a = \frac{-(-2)}{2(1)} = 1$
- 8.(A)** $x = \sqrt{7+4\sqrt{3}} = \sqrt{(2+\sqrt{3})^2} = 2+\sqrt{3}$
 $x + \frac{1}{x} = (2+\sqrt{3}) + \frac{1}{(2+\sqrt{3})} = (2+\sqrt{3}) + (2-\sqrt{3}) = 4$
- 9.(C)** Let the roots be α, α^2
 Then $\alpha^2 + \alpha = 1$ and $(\alpha)(\alpha^2) = \alpha^3 = -k$ or $\alpha^2 + \alpha - 1 = 0$
 $\Rightarrow \alpha = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow \alpha^3 = \frac{(-1 \pm \sqrt{5})^3}{8} = \frac{(-1 \pm 5\sqrt{5} \pm 3\sqrt{5} - 15)}{8} = -2 \pm \sqrt{5} = -k$ or $k = 2 \mp \sqrt{5}$
- 10.(C)** $P(x) = (x-1)(x+2)(ax+b)$
 $P(-1) = 4 \Rightarrow -2(b-a) = 4 \Rightarrow a-b = 2$
 $P(2) = 28 \Rightarrow 4(2a+b) = 28 \Rightarrow 2a+b = 7$
 On solving $a = 3, b = 1$
- 11.(A)** Remainder $= P(-3) = -32$

12.(A) $\alpha - \beta = 1$

$$p^2 - 4q = 1$$

$$p^2 - 4q + 4q^2 = 1 + 4q^2$$

$$p^2 + 4q^2 = (1 + 2q)^2$$

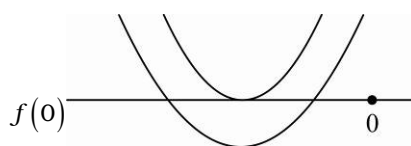
13.(D) $x^2 + 2(a+1)x + 9a - 5 = 0$

For both roots to be -ve

Desired graph is

$$D \geq 0$$

$$\frac{-b}{2a} < 0$$



$$(a+1)^2 - 9a + 5 \geq 0$$

$$a \in (-\infty, 1] \cup [6, \infty)$$

$$\frac{-b}{2a} < 0$$

Given

$$-(a+1) < 0$$

$$a+1 > 0$$

$$a > -1$$

$$f(0) > 0$$

$$9a - 5 > 0$$

$$a > \frac{5}{9} \quad \therefore \quad \left(\frac{5}{9}, 1\right] \cup [6, \infty)$$

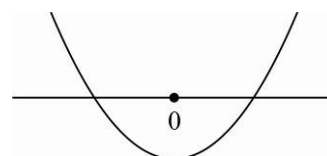
is the final solution.

14.(C) $3x^2 + 2(a^2 + 1)x + a^2 - 3a + 2 = 0$

For roots to be of opposite signs derived graph is $f(0) < 0$

$$a^2 - 3a + 2 < 0 \Rightarrow (a-1)(a-2) < 0$$

$$a \in (1, 2)$$



15.(A) $(x-m)^2 = 1 \Rightarrow x = m+1, m-1$

Case 1: Both roots lie in $(-2, 4)$

So, $m-1 > -2$ and $m+1 < 4$

$$m \in (-1, 3)$$

Case 2:

Exactly one root lie in $(-2, 4)$

$$\text{if } (m-1) \in (-2, 4) \Rightarrow m+1 \geq 4 \Rightarrow m \in [3, 5)$$

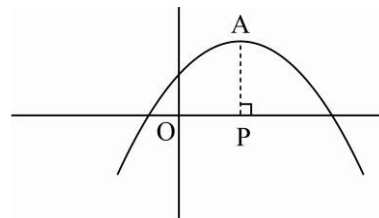
and if

$$\begin{aligned}(m+1) &\in (-2, 4) \Rightarrow m-1 \leq -2 \Rightarrow m \in (-3, -1] \\ \Rightarrow m &\in (-3, -1] \cup (-1, 3) \cup [3, 5) \\ \Rightarrow m &\in (-3, 5)\end{aligned}$$

16.(C) Equation of curve $y = ax^2 + bx + c$

$$AP = \frac{-(b^2 - 4ac)}{4a} = 3 \Rightarrow \frac{-2b}{4a} = 3$$

$$\text{So } \frac{-b}{2a} = 3 \Rightarrow OP = 3$$



17.(D) $x^2 - x \ln(a^2 - 3a + 2) + a^2 - 4 = 0$

For defining logarithm

$$a^2 - 3a + 2 > 0$$

$$(a-1)(a-2) > 0$$

$$a \in (-\infty, 1) \cup (2, \infty) \quad \dots \text{(i)}$$

Roots are of opposite sign so

$$f(0) = a^2 - 4 < 0$$

$$a \in (-2, 2) \quad \dots \text{(ii)}$$

From (i) and (ii)

$$a \in (-2, 1)$$

18.(D) Since, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$$\text{Also, } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2 - 2ac}{a^2}$$

$$\text{Now, } \frac{\alpha}{\alpha\beta + b} + \frac{\beta}{\alpha\alpha + b} = \frac{\alpha(\alpha^2 + \beta^2) + b(\alpha + \beta)}{\alpha\beta\alpha^2 + ab(\alpha + \beta) + b^2} = \frac{a\left(\frac{b^2 - 2ac}{a^2}\right) + b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2 + ab\left(-\frac{b}{a}\right) + b^2} = -\frac{2}{a}$$

19.(D) Let $y = \frac{x+2}{2x^2+3x+6} \Rightarrow 2yx^2 + (3y-1)x + 6y-2 = 0$

But x is real, then $(3y-1)^2 - 4(2y)(6y-2) \geq 0 \quad [\because D \geq 0]$

$$\Rightarrow (13y+1)(3y-1) \leq 0 \Rightarrow -\frac{1}{13} \leq y \leq \frac{1}{3}$$

20.(A) Let α and β be the roots of equation $x^2 - (a-2)x - a - 1 = 0$

Then, $\alpha + \beta = a - 2$ and $\alpha\beta = -a - 1$

$$\alpha^3 + \beta^3 = (a-2)^3 + 3(a+1)(a-2)$$

The value of $\alpha^3 + \beta^3$ will be least, if $a - 2 = 0$

21.(B) Given that, $3^{2x^2-7x+7} = 3^2 \Rightarrow 2x^2 - 7x + 7 = 2 \Rightarrow 2x^2 - 7x + 5 = 0$

$$\text{Now, } D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 5 = 49 - 40 = 9 > 0$$

Hence, it has two-real roots.

22.(C) Since, $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ (i)

So, $(1-p)$ satisfied the above equation. $\therefore (1-p)^2 + p(1-p) + (1-p) = 0$

$$\Rightarrow (1-p)(1-p+p+1) = 0 \Rightarrow (1-p)2 = 0 \Rightarrow p = 1$$

On putting this value in equation (i), we get : $x^2 + x = 0 \Rightarrow x = 0, -1$

23.(B) Given equation $x^2 - 3|x| + 2 = 0$

Case I : When $x > 0$, then $|x| = x \quad \therefore x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0 \Rightarrow x = 1, 2$

Case II : When $x < 0$, then $|x| = -x \quad \therefore x^2 + 3x + 2 = 0 \Rightarrow (x+1)(x+2) = 0 \Rightarrow x = -1, -2$

Hence, four solutions are possible.

24.(C) $2^{(x-1)(x^2+5x-50)} = 1$

$$\Rightarrow (x-1)(x^2+5x-50) = 0 \Rightarrow x = 1 \text{ or } x^2 + 5x - 50 = 0$$

Sum of all real values $= 1 + (-5) = -4$

25.(C) $p(0) = 1$

$p(x)$ leave remainder 4 when divided by $(x-1)$

$$\Rightarrow p(x) = 4$$

Similarly $p(-1) = 6$

Assume $p(1) = ax^2 + bx + c$

$$p(0) = 1 \Rightarrow c = 1$$

$$p(1) = a + b + 1 = 4$$

$$(-1) = a - b + 1 = 6$$

$$\Rightarrow a = 4, b = -1$$

$$p(x) = 4x^2 - x + 1$$

$$p(2) = 4 \times 4 - 2 + 1 = 15$$

$$p(-2) = 16 + 2 + 1 = 19$$

26.(B) Given equations are $2x^2 + 3x + 4 = 0$ (i) and $ax^2 + bc + c = 0$ (ii)

Since, equation (i) has imaginary roots so, equation (ii) will also have both roots same as equation (i)

$$\text{Thus, } \frac{a}{2} = \frac{b}{3} = \frac{c}{4} \quad \text{Hence } a : b : c \text{ is } 2 : 3 : 4$$

27.(C) $x^2 + |2x-3| - 4 = 0$

Case 1 : $x \geq \frac{3}{2}$

$$x^2 + 2x - 7 = 0$$

$$x = \frac{-2 \pm \sqrt{32}}{2} \quad x = -1 + 2\sqrt{2}, -1 - 2\sqrt{2} < \frac{3}{2} \text{ (Reject)}$$

Case 2: $x < \frac{3}{2}$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = 1 \pm \sqrt{2}$$

$$x = 1 - \sqrt{2}, 1 + \sqrt{2} > \frac{3}{2} \text{ (Reject)}$$

$$\text{Sum of roots} = -1 + 2\sqrt{2} + 1 - \sqrt{2} = \sqrt{2}$$

28.(A)

$$\sqrt{3x^2 + x + 5} = x - 3$$

Squaring $x \geq 3$

$$3x^2 + x + 5 = x^2 - 6x + 9$$

$$x^2 + 7x - 4 = 0$$

$$2x^2 + 8x - x - 4 = 0$$

$$2x(x + 4) - 1(x + 4) = 0$$

$$x = -4 \text{ or } x = \frac{1}{2}$$

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MEQB

Level - 2

29.(A) Let two consecutive integers n and $(n + 1)$ be the roots of $x^2 - bx + c = 0$. Then, $n + (n + 1) = b$ and $n(n + 1) = c$ $\therefore b^2 - 4c = (2n + 1)^2 - 4n(n + 1) = 1$

30.(A) Let $f(x) = x^{2n} - 1$

At $x = \pm 1$, $f(x) = 0$ Hence, for no other real value of x , $f(x)$ is zero.

31.(A) α, β are the roots of $ax^2 + bx + c = 0 \Rightarrow a\alpha^2 + b\alpha + c = 0, a\beta^2 + b\beta + c = 0$

$$\Rightarrow \alpha(a\alpha + b) = -c, \beta(a\beta + b) = -c \Rightarrow a\alpha + b = -\frac{c}{\alpha}, a\beta + b = -\frac{c}{\beta}$$

$$\left(\frac{\alpha}{a\beta + b}\right)^3 - \left(\frac{\beta}{a\alpha + b}\right)^3 = \left(\frac{\alpha}{-c/\beta}\right)^3 - \left(\frac{\beta}{-c/\alpha}\right)^3 = \frac{-(\alpha\beta)^3}{c^3} + \frac{(\alpha\beta)^3}{c^3} = 0$$

32.(B) $\alpha^2, -\beta^2$ are the roots of $\alpha^2 x^2 + x + 1 - \alpha^2 = 0 \Rightarrow \alpha^2 - \beta^2 = -\frac{1}{\alpha^2}, \alpha^2(-\beta^2) = \frac{1 - \alpha^2}{\alpha^2}$

$$(\alpha^2 + \beta^2)^2 = (\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2 = \frac{1}{\alpha^4} + 4\left(\frac{\alpha^2 - 1}{\alpha^2}\right) = \frac{1}{\alpha^4} + 4 - \frac{4}{\alpha^2} = \left(2 - \frac{1}{\alpha^2}\right)^2 \Rightarrow \alpha^2 + \beta^2 = 2 - \frac{1}{\alpha^2}$$

$$\beta^2 = \frac{1}{2} \left[(\alpha^2 + \beta^2) - (\alpha^2 - \beta^2) \right] = \frac{1}{2} \left[2 - \frac{1}{\alpha^2} + \frac{1}{\alpha^2} \right] = 1$$

33.(D) From the given equation $\tan 30^\circ + \tan 15^\circ = -p$ and $\tan 30^\circ \tan 15^\circ = q$

$$\text{Now } \tan(30^\circ + 15^\circ) = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ} \Rightarrow 1 = -\frac{p}{1 - q} \Rightarrow 1 - q = -p \Rightarrow q - p = 1 \Rightarrow 2 + q - p = 3$$

34.(D) Since roots are real, we have $\cos p - 1 \neq 0$ and discriminant $\geq 0 \therefore \cos^2 p - 4(\cos p - 1)\sin p \geq 0$

$$\Rightarrow \cos^2 p + 4(1 - \cos p)\sin p \geq 0 \Rightarrow \sin p \geq 0 \quad (\because \cos p - 1 \neq 0, -1 \leq \cos p \leq 1 \Rightarrow 1 - \cos p > 0)$$

$$\Rightarrow p \in [0, \pi] \Rightarrow p \in (0, \pi) \quad (\because \cos p \neq 1)$$

35.(D) We have $(b - c)x^2 + (c - a)x + (a - b) = 0 \quad \dots \text{(i)}$

a, b, c are in A.P. Let common difference be k .

$$\therefore \text{(i)} \Rightarrow -kx^2 + 2kx - k = 0 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1, 1$$

$$\therefore 1 \text{ is a root of } 2(c + a)x^2 + (b + c)x = 0 \Rightarrow 2(c + a) + (b + c) = 0 \Rightarrow 2a + b + 3c = 0$$

$$a, b, c \text{ are in A.P.} \Rightarrow a - 2b + c = 0 \therefore \frac{a}{1 + 6} = \frac{b}{3 - 2} = \frac{c}{-4 - 1} \Rightarrow \frac{a}{7} = \frac{b}{1} = \frac{c}{-5} = \lambda \text{ (say)}$$

$$\Rightarrow a = 7\lambda, b = \lambda, c = -5\lambda \Rightarrow a^2 = 49\lambda^2, b^2 = \lambda^2, c^2 = 25\lambda^2 \Rightarrow a^2 + b^2 = 2c^2 \Rightarrow a^2, c^2, b^2 \text{ are in A.P.}$$

36.(C) $(2x + 3)(2x + 5)(x - 1)(x - 2) = 30 \Rightarrow [(2x + 3)(x - 1)][(2x + 5)(x - 2)] = 30$

$$\Rightarrow (2x^2 + x - 3)(2x^2 + x - 10) = 30 \Rightarrow (y - 3)(y - 10) = 30 \text{ where } y = 2x^2 + x$$

$$\Rightarrow y^2 - 13y = 0 \Rightarrow y = 0 \text{ or } 13 \Rightarrow 2x^2 + x = 0 \text{ or } 2x^2 + x = 13 \Rightarrow x = 0$$

$$\text{or } -\frac{1}{2} \text{ or } x = \frac{-1 \pm \sqrt{1+104}}{4} \quad \therefore \quad \text{The number of rational roots} = 2.$$

37.(C) Note that $f(a) = a^2$, $f(b) = b^2$ and $f(c) = c^2$

If $g(x) = f(x) - x^2$, then $g(a) = g(b) = g(c) = 0$

That is, $f(x) - x^2$ is quadratic expression which vanishes at three distinct points a , b and c .

Thus, $f(x) - x^2 \equiv 0$ or $f(x) \equiv x^2$

38.(C) Let $y = \frac{x-a}{x^2-3x+2}$. Then $yx^2 - 3yx + 2y = x - a \Rightarrow yx^2 + (-3y-1)x + (2y+a) = 0$

$$x \text{ is real} \Rightarrow (-3y-1)^2 - 4y(2y+a) \geq 0 \Rightarrow 9y^2 + 6y + 1 - 8y^2 - 4ay \geq 0 \Rightarrow y^2 + (6-4a)y + 1 \geq 0$$

Since y is real, $y^2 + (6-4a)y + 1 \geq 0 \Rightarrow$ The roots of $y^2 + (6-4a)y + 1 = 0$ are real and equal or imaginary

$$\Rightarrow \text{Discriminant} \leq 0 \Rightarrow (6-4a)^2 - 4 \leq 0 \Rightarrow 36 + 16a^2 - 48a - 4 \leq 0 \Rightarrow 16a^2 - 48a + 32 \leq 0$$

$$16a^2 - 48a + 32 = 0 \Rightarrow a^2 - 3a + 2 = 0 \Rightarrow (a-1)(a-2) = 0 \Rightarrow a = 1$$

$$\text{or } 2 \quad 16a^2 - 48a + 32 \leq 0 \Rightarrow 1 \leq a \leq 2$$

39.(B) Remainders of $f(x)$ when divided by $x-1$, $x-2$, $x-3$ are $f(1)$, $f(2)$, $f(3)$ respectively.

Then $f(1) = 3$, $f(2) = 7$, $f(3) = 13$

Since $(x-1)(x-2)(x-3)$ is a third-degree expression, the remainder of $f(x)$ when divided by

$$(x-1)(x-2)(x-3) \text{ is of second degree} \quad \therefore f(x) = q(x-1)(x-2)(x-3) + ax^2 + bx + c$$

$$f(1) = 3 \Rightarrow a + b + c = 3 \quad \dots \text{(i)}, \quad f(2) = 7 \Rightarrow 4a + 2b + c = 7 \quad \dots \text{(ii)}$$

$$f(3) = 13 \Rightarrow 9a + 3b + c = 13 \quad \dots \text{(iii)}$$

$$\text{(ii)} - \text{(i)} \Rightarrow 3a + b = 4 \quad \dots \text{(iv)}$$

$$\text{(iii)} - \text{(ii)} \Rightarrow 5a + b = 6 \quad \dots \text{(v)}$$

$$\text{(v)} - \text{(iv)} \Rightarrow 2a = 2 \Rightarrow a = 1 \quad \therefore b = 1, c = 1 \quad \therefore \text{Remainder} = x^2 + x + 1$$

40.(C) $x^2 + 4y^2 - 8x + 12 = 0 \Rightarrow x^2 - 8x + (4y^2 + 12) = 0 \quad \dots \text{(ii)}$

$$\text{(ii) is a quadratic in } x, x \text{ is real} \Rightarrow \text{Discriminant} \geq 0 \Rightarrow (-8)^2 - 4(4y^2 + 12) \geq 0$$

$$\Rightarrow -16y^2 + 16 \geq 0 \Rightarrow y^2 - 1 \leq 0 \Rightarrow -1 \leq y \leq 1 \quad \therefore y \text{ must lie between } -1 \text{ and } 1.$$

41.(C) $\left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x \geq 1 \quad \therefore \cos^x \alpha + \sin^x \alpha \geq 1, \text{ where } \cos \alpha = \frac{5}{13}$

Equality holds for $x = 2$. If $x < 2$, both $\cos \alpha$ and $\sin \alpha$ increase (being positive fractions)

So, $\cos^x \alpha + \sin^x \alpha > 1$ if $x < 2$.

42.(D) $y = \frac{2a(x-1)\sin^2 \alpha}{x^2 - \sin^2 \alpha} \Rightarrow yx^2 - y\sin^2 \alpha = 2ax\sin^2 \alpha - 2a\sin^2 \alpha \Rightarrow yx^2 - 2ax\sin^2 \alpha + (2a - y)\sin^2 \alpha = 0$

$$x \in R \Rightarrow 4a^2 \sin^4 \alpha - 4y(2a - y)\sin^2 \alpha \geq 0 \Rightarrow a^2 \sin^2 \alpha - 2ay + y^2 \geq 0 \Rightarrow y^2 - 2ay + a^2 \sin^2 \alpha \geq 0$$

$$\text{Let } y^2 - 2ay + a^2 \sin^2 \alpha = 0 \Rightarrow y = \frac{2a \pm \sqrt{4a^2 - 4a^2 \sin^2 \alpha}}{2} \Rightarrow y = a \pm a \cos \alpha = a(1 \pm \cos \alpha)$$

$$\Rightarrow y = 2a \sin^2 \frac{\alpha}{2} \text{ or } 2a \cos^2 \frac{\alpha}{2}$$

$$\therefore y^2 - 2ay + a^2 \sin^2 \alpha \geq 0 \Rightarrow y \text{ cannot lie between } 2a \sin^2 \frac{\alpha}{2}, 2a \cos^2 \frac{\alpha}{2}$$

43.(A) $ax + by = 1 \Rightarrow by = 1 - ax \Rightarrow y = \frac{(1 - ax)}{b}$

$$cx^2 + dy^2 = 1 \Rightarrow cx^2 + d \frac{(1 - ax)^2}{b^2} = 1 \Rightarrow b^2 cx^2 + d(1 + a^2 x^2 - 2ax) = b^2$$

$$\Rightarrow (b^2 c + a^2 d)x^2 - 2adx + d - b^2 = 0$$

$$\text{Both the equations have only one solution} \Rightarrow \text{Discriminant} = 0 \Rightarrow 4a^2 d^2 - 4(b^2 c + a^2 d)(d - b^2) = 0$$

$$\Rightarrow a^2 d^2 - (b^2 c + a^2 d)(d - b^2) = 0 \Rightarrow a^2 d^2 - b^2 cd + b^4 c - a^2 d^2 + a^2 b^2 d = 0 \Rightarrow b^4 c + a^2 b^2 d = b^2 cd$$

$$\Rightarrow \frac{b^2}{d} + \frac{a^2}{c} = 1$$

44.(C) $0 < \alpha < \beta < \gamma < \frac{\pi}{2} \Rightarrow \sin \alpha < \sin \beta < \sin \gamma$

$$\text{Now, the given equation is } (x - \sin \beta)(x - \sin \gamma) + (x - \sin \alpha)(x - \sin \gamma) + (x - \sin \alpha)(x - \sin \beta) = 0$$

$$\text{Let } f(x) = (x - \sin \beta)(x - \sin \gamma) + (x - \sin \alpha)(x - \sin \gamma) + (x - \sin \alpha)(x - \sin \beta) = 0$$

$$\Rightarrow f(\sin \alpha) = (\sin \alpha - \sin \beta)(\sin \alpha - \sin \gamma) > 0 \Rightarrow f(\sin \beta) = (\sin \beta - \sin \alpha)(\sin \beta - \sin \gamma) < 0$$

$$\Rightarrow f(\sin \gamma) = (\sin \gamma - \sin \alpha)(\sin \gamma - \sin \beta) > 0$$

Hence, equation $f(x) = 0$ has 1 root between $\sin \alpha$ and $\sin \beta$ and other root between $\sin \beta$ and $\sin \gamma$.

45.(B) α, β are roots of $x^2 + px + q = 0 \Rightarrow \alpha + \beta = -p$ and $\alpha\beta = q$

$$\alpha, \beta \text{ are also roots of } x^{2n} + p^n x^n + q^n = 0 \Rightarrow \alpha^{2n} + p^n \alpha^n + q^n = 0 \quad \dots \text{(i)}$$

$$\beta^{2n} + p^n \beta^n + q^n = 0 \quad \dots \text{(ii)}$$

$$(i) - (ii) \Rightarrow \alpha^{2n} - \beta^{2n} + p^n (\alpha^n - \beta^n) = 0 \Rightarrow (\alpha^n - \beta^n) (\alpha^n + \beta^n + p^n) = 0 \Rightarrow \alpha^n + \beta^n = -p^n \quad \dots \text{(iii)}$$

$$\frac{\alpha}{\beta} \text{ and } \frac{\beta}{\alpha} \text{ are roots of } x^n + 1 + (x+1)^n = 0 \Rightarrow \frac{\alpha^n}{\beta^n} + 1 + \left(\frac{\alpha}{\beta} + 1\right)^n = 0 \Rightarrow \alpha^n + \beta^n + (\alpha + \beta)^n = 0$$

$$\Rightarrow -p^n + (-p)^n = 0 \Rightarrow p^n [-1 + (-1)^n] = 0 \text{ This is possible only when } n \text{ is even.}$$

46.(C) If α is the common root then $(1 - 2a)\alpha^2 - 6a\alpha - 1 = 0 \quad \dots \text{(i)}$

$$\alpha\alpha^2 - \alpha + 1 = 0 \quad \dots \text{(ii)}$$

$$\begin{array}{ccccc}
 \alpha^2 & & \alpha & & 1 \\
 -6a & \nearrow & -1 & \nearrow & 1-2a & \nearrow & -6a \\
 -1 & \searrow & 1 & \searrow & a & \searrow & -1
 \end{array}$$

$$\frac{\alpha^2}{-6a-1} = \frac{\alpha}{-a-1+2a} = \frac{1}{-1+2a+6a^2} \Rightarrow \alpha^2 = \frac{-6a-1}{6a^2+2a-1}, \alpha = \frac{a-1}{6a^2+2a-1}$$

$$\Rightarrow \frac{(a-1)^2}{(6a^2+2a-1)^2} = \frac{-6a-1}{6a^2+2a-1} \Rightarrow (a-1)^2 = (-6a-1)(6a^2+2a-1)$$

$$\Rightarrow a^2 - 2a + 1 = -36a^3 - 12a^2 + 6a - 6a^2 - 2a + 1 \Rightarrow 36a^3 + 19a^2 - 6a = 0$$

$$\Rightarrow a(36a^2 + 19a - 6) = 0 \Rightarrow a(4a+3)(9a-2) = 0 \Rightarrow a = \frac{2}{9}, \frac{-3}{4}$$

47.(A) p, q, r are in A.P. $\Rightarrow p+r=2q$

The roots of $px^2+qx+r=0$ are real $\Rightarrow q^2-4pr \geq 0 \Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0 \Rightarrow p^2+r^2-14pr \geq 0$

$$\Rightarrow 1 + \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) \geq 0 \Rightarrow \left(\frac{r}{p} - 7\right)^2 - 49 + 1 \geq 0 \Rightarrow \left(\frac{r}{p} - 7\right)^2 \geq 48 \Rightarrow \left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$$

48.(AC) α, β are the roots of $x^2+px+q=0 \Rightarrow \alpha+\beta=-p, \alpha\beta=q$

α^4, β^4 are the roots of $x^2-rx+s=0 \Rightarrow \alpha^4+\beta^4=r, \alpha^4\beta^4=s$

Discriminant of $x^2-4qx+2q^2-r=0$ is $(4q)^2-4(2q^2-r)$

$$= 16q^2 - 8q^2 + 4r = 8q^2 + 4r = 8\alpha^2\beta^2 + 4(\alpha^4 + \beta^4) = 4(\alpha^4 + \beta^4 + 2\alpha^2\beta^2) = 4(\alpha^2 + \beta^2)^2 > 0$$

49.(A) Since, $\sin A, \sin B, \cos A$ are in GP.

$\therefore \sin^2 B = \sin A \cos A \quad \dots \text{(i)}$

Also, $x^2 + 2x \cot B + 1 = 0$ [Given]

$$\text{Now, } b^2 - 4ac = 4 \cot^2 B - 4 = \frac{4 \cos^2 B - 4 \sin^2 B}{\sin^2 B} = \frac{4(1 - 2 \sin^2 B)}{\sin^2 B}$$

$$= \frac{4(1 - 2 \sin A \cos A)}{\sin^2 B} = 4 \left(\frac{\sin A - \cos A}{\sin B} \right)^2 \geq 0 \quad [\text{Using (i)}]$$

$\Rightarrow$ Roots of given equation are always real.

50.(A) Since, a, b and c are the sides of a $\triangle ABC$, then : $|a-b| < |c| \Rightarrow a^2 + b^2 - 2ab < c^2$

Similarly, $b^2 + c^2 - 2bc < a^2, c^2 + a^2 - 2ca < b^2$

On adding, we get : $(a^2 + b^2 + c^2) < 2(ab + bc + ca) \Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} < 2$

Also, $D \geq 0, (a+b+c)^2 - 3\lambda(ab+bc+ca) \geq 0 \Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} > 3\lambda - 2 \quad \dots \text{(ii)}$

From equations (i) and (ii), $3\lambda - 2 < 2 \Rightarrow \lambda < \frac{4}{3}$

51.(C) Let $\frac{x^2 + 2x - 11}{x - 3} = m$

$$\Rightarrow x^2 + 2x - 11 = mx - 3m \Rightarrow x^2 + (2 - m)x + (3m - 11) = 0 \quad \dots\dots\dots(i)$$

Since x is real, therefore, discriminant of equation (i) must be non-negative.

i.e. $(2 - m)^2 - 4 \times 1 \times (3m - 11) \geq 0$

$$\Rightarrow 4 - 4m + m^2 - 12m + 44 \geq 0 \Rightarrow m^2 - 16m + 48 \geq 0$$

$$\Rightarrow (m - 4)(m - 12) \geq 0 \Rightarrow m \leq 4 \text{ or } m \geq 12$$

52.(C) $a + b + c = 0$; $a^2 + b^2 + c^2 = 1$

$$a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)^2 - 2(a^2b^2 + b^2c^2 + c^2a^2) = 1 - 2(a^2b^2 + b^2c^2 + c^2a^2) \quad \dots\dots(i)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$0 = 1 + 2(ab + bc + ca) \Rightarrow ab + bc + ca = \frac{-1}{2}$$

$$(ab + bc + ca)^2 = a^2b^2 + b^2c^2 + c^2a^2 + 2(ab^2c + abc^2 + bca^2)$$

$$\frac{1}{4} = (a^2b^2 + b^2c^2 + c^2a^2) + 2abc(a + b + c)$$

$$a^2b^2 + b^2c^2 + c^2a^2 = \frac{1}{4} \quad \text{Put in (i), we get :}$$

$$a^4 + b^4 + c^4 = 1 - 2 \times \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$a^4 + b^4 + c^4 = \frac{1}{2}$$

53.(D) $2\{(x - p)(x - q) + (p - q)(p - x - q + x)\} = (p - q)^2 + (x - p)^2 + (x - q)^2$

$$\Rightarrow 2(x - p)(x - q) + (p - q)^2 = (x - p)^2 + (x - q)^2$$

$$\Rightarrow 2x^2 - 2x(p + q) + 2pq + p^2 + q^2 - 2pq = 2x^2 + p^2 + q^2 - 2x(p + q)$$

It is an identity so infinite values of x are possible.

Solution to 54 to 56

54.(A) 55.(B) 56.(C)

$$\frac{-p}{4} = -3, \frac{-(p^2 - 8q)}{8} = 1$$

$$p = 12, 8q - p^2 = 8$$

$$8q - 144 = 8$$

$$q = \frac{152}{8}$$

$$q = 19$$

$$f(x) = 2x^2 + 12x + 19$$

$$D = 144 - 152 < 0$$

57.(D) $\therefore m > 0$ and $D < 0$
 $16 - 4m(3m + 1) < 0$
 $4 - m(3m + 1) < 0$
 $3m^2 + m - 4 > 0$
 $(3m + 4)(m - 1) > 0$
 $m \in \left(-\infty, -\frac{4}{3}\right) \cup (1, \infty)$

But $m > 0$

$\therefore m \in (1, \infty)$

Least integral value = 2

58.(B) $(a - 6)x^2 - 2 = 2ax + 1 \Rightarrow (a - 6)x^2 - 2ax - 3 = 0$
 $D < 0$
 $4a^2 + 12(a - 6) < 0$
 $a^2 + 3a - 18 < 0$
 $(a + 6)(a - 3) < 0$
 $a \in (-6, 3)$

Least integer = -5

59.(C) i.e. $\alpha x^2 + 6x - 8 = 0$ (i)
 and $-8x^2 + 6x + \alpha = 0$ (ii)

Have a common root.

Solving (i) and (ii) for common root

(i) - (ii)

$(\alpha + 8)x^2 = (8 + \alpha) \Rightarrow x = \pm 1$

So, from (i) $\alpha = 2, 14$

If both roots are common $\frac{\alpha}{-8} = 1 \Rightarrow \alpha = -8$ So $\alpha = \{2, 14, -8\}$

60.(C) $3^{2(x^2-x-6)} - 2 \cdot 3^{(x^2-x-6)} + 1 = 0$
 $\Rightarrow 3^{(x^2-x-6)} = 1 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x - 3)(x + 2) = 0 \Rightarrow x = -2, 3$

61.(A) Given, $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$
 $\Rightarrow \log_{10} \frac{(x^3 + y^3)}{x^2 + y^2 - xy} \leq 2 \Rightarrow \log_{10}(x + y) \leq 2 \Rightarrow x + y \leq 100$

Using $AM \geq GM$

$\therefore \frac{x + y}{2} \geq \sqrt{xy} \Rightarrow \sqrt{xy} \leq \frac{x + y}{2} \leq \frac{100}{2} \Rightarrow xy \leq 2500$

62.(A) Let α be the common root of $x^2 + px + qr = 0$ and $x^2 + qx + rp = 0$.

Then $\alpha^2 + p\alpha + qr = 0 \quad \dots \text{.(i)}$ and $\alpha^2 + q\alpha + rp = 0 \quad \dots \text{.(ii)}$

Subtracting (ii) from (i), we get :

$$(p - q)\alpha + r(q - p) = 0 \quad \text{or} \quad (p - q)(\alpha - r) = 0 \quad \Rightarrow \quad \alpha = r \quad (\text{Since } p \neq q).$$

Similarly, p and q are other two common roots.

$$\Rightarrow \quad \text{Product of common roots} = pqr$$

63.(D) Let, $f(x) = (x - a_1)^2 + (x - a_2)^2 + \dots + (x - a_n)^2$
 $= nx^2 - 2(a_1 + a_2 + \dots + a_n)x + (a_1^2 + a_2^2 + \dots + a_n^2)$

$$f(x) \text{ assumes least value when } x = \frac{-[-2(a_1 + a_2 + \dots + a_n)]}{2n} = \frac{a_1 + a_2 + \dots + a_n}{n}$$

64.(AB) Let α be a root of $x^2 - x + m = 0$. Then 2α is a root of $x^2 - 3x + 2m = 0$

$$\Rightarrow \quad \alpha^2 - \alpha + m = 0 \quad \text{and} \quad 4\alpha^2 - 6\alpha + 2m = 0 \quad \text{or} \quad 2\alpha^2 - 3\alpha + m = 0 \quad \dots \text{.(ii)}$$

$$\text{or} \quad 2\alpha^2 - 2\alpha + 2m = 0 \quad \dots \text{.(i)}$$

$$\text{Subtracting (ii) from (i), we get :} \quad \alpha + m = 0 \quad \text{or} \quad \alpha = -m$$

$$\text{Putting value of } \alpha \text{ in (i), we get :} \quad 2m^2 + 4m = 0 \quad \text{or} \quad m = 0, -2$$

65.(C) α, β are roots of the equation $(x - a)(x - b) + c = 0$ or $x^2 - (a + b)x + ab + c = 0$

$$\Rightarrow \quad \alpha + \beta = a + b \quad \dots \text{.(i)}$$

$$\text{and} \quad \alpha\beta = ab + c \quad \dots \text{.(ii)}$$

Let p, q be roots of the equation $(x - c - \alpha)(x - c - \beta) = c$

$$\text{or} \quad x^2 - (\alpha + \beta + 2c)x + c^2 + c(\alpha + \beta - 1) + \alpha\beta = 0$$

$$\Rightarrow \quad p + q = \alpha + \beta + 2c \quad \dots \text{.(iii)} \quad \text{and} \quad pq = c^2 + c(\alpha + \beta - 1) + \alpha\beta \quad \dots \text{.(iv)}$$

Putting value of $\alpha + \beta$ from (i) into (iii), (iv) and value of $\alpha\beta$ from (ii) into (iv), we get :

$$p + q = a + b + 2c = (a + c) + (b + c) \quad \text{and} \quad pq = c^2 + c(a + b - 1) + ab + c = (a + c) \cdot (b + c)$$

$$\Rightarrow \quad p = a + c \quad \text{and} \quad q = b + c$$

66.(D) $2e^{2 \log k} - 1 = 31$ or $2e^{2 \log k} = 32$ or $e^{2 \log k} = 16$

$$\text{or} \quad k^2 = 16 \quad \text{or} \quad k = \pm 4.$$

67.(B) $y = \frac{x^2 + 2x + a}{x^2 + 4x + 3a}$ gives $(y - 1)x^2 + (4y - 2)x + (3ay - a) = 0$

Since x is real $D \geq 0$,

$$(4y - 2)^2 - 4(y - 1)(3ay - a) \geq 0 \quad \text{or} \quad (4y^2 - 4y + 1) - a(y - 1)(3y - 1) \geq 0$$

$$\text{or} \quad (4 - 3a)y^2 + (4a - 4)y + (1 - a) \geq 0$$

$$\text{Since } y \in R \quad 4 - 3a > 0 \quad \text{and} \quad (4a - 4)^2 - 4(4 - 3a)(1 - a) \leq 0$$

$$\text{or} \quad a < \frac{4}{3} \quad \text{and} \quad 16(a^2 - 2a + 1) - 4(3a^2 - 7a + 4) \leq 0$$

$$\Rightarrow a < \frac{4}{3} \text{ and } 4a^2 - 4a \leq 0 \Rightarrow a < \frac{4}{3} \text{ and } a(a-1) \leq 0$$

$$\Rightarrow a < \frac{4}{3} \text{ and } a \in [0, 1] \Rightarrow a \in [0, 1]$$

68.(C) Determinant of $f(x) = D_1 = b^2 - 4ac$ and Determinant of $g(x) = D_2 = b^2 + 4ac$

$$D_1 + D_2 = 2b^2 \geq 0 \Rightarrow \text{Atleast one of } D_1 \text{ and } D_2 \geq 0$$

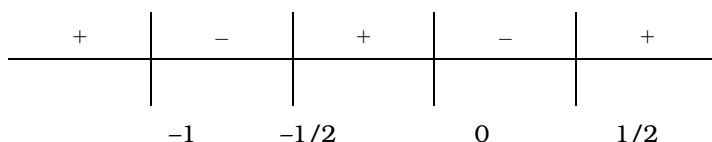
$$\Rightarrow f(x) \cdot g(x) = 0 \text{ has at least two-real roots.}$$

69.(D) $(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - (\gamma + \delta)\alpha + \gamma\delta$

Since γ, δ are roots of $x^2 + px + r = 0$, $\gamma + \delta = -p$ and $\gamma\delta = r \Rightarrow (\alpha - \gamma)(\alpha - \delta) = \alpha^2 + p\alpha + r$

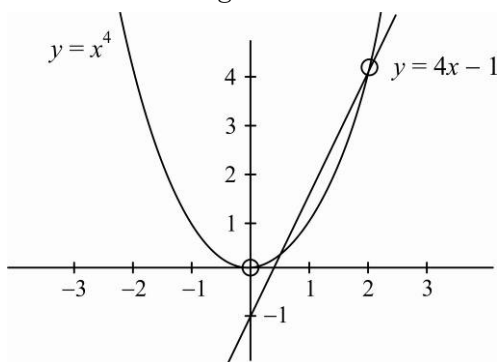
Since α is a root of $x^2 + px - q = 0$, $\alpha^2 + p\alpha - q = 0$ or $\alpha^2 + p\alpha = q \Rightarrow (\alpha - \gamma)(\alpha - \delta) = q + r$

70.(AD) $\frac{2x-1}{2x^3+3x^2+x} > 0$ gives $\frac{2x-1}{x(x+1)(2x+1)} > 0$



Solution : $x \in (-\infty, -1) \cup \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

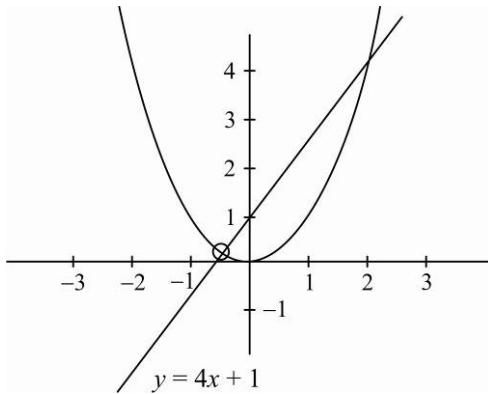
71.(B) $x^4 - 4x + 1 = 0$ gives $x^4 = 4x - 1$



From the figure it is clear that the given equation has 2 positive real roots.

72.(C) $x^4 - 4x - 1 = 0$ gives $x^4 = 4x + 1$

From the graph it is clear that the given equation has one negative real root.



73.(B) From the above graph we see that there are 2-real roots for the equation $x^4 - 4x - 1 = 0$. The equation has 4 roots. Hence remaining 2 roots are complex roots.

74.(A) For $x^2 + x + 1 = 0$, $D < 0$. So, it has non-real roots.

As $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d$, roots of $x^2 + x + 1 = 0$ are also roots of $ax^3 + bx^2 + cx + d = 0$

$\Rightarrow ax^3 + bx^2 + cx + d = 0$ has 2 non-real roots.

Let $x = p$ be the remaining real root. Product of roots $\frac{-d}{a} = p$ [Product of roots of $x^2 + x + 1 = 0$]

75.(C) $x^3 - px^2 + qx - r = 0$

α, β, γ are roots of equation $\sum \alpha^2\beta = \alpha^2\beta + \alpha^2\gamma + \beta^2\alpha + \beta^2\gamma + \gamma^2\alpha + \gamma^2\beta$

$$= (\alpha^2\beta + \alpha^2\gamma + \alpha\beta\gamma) + (\beta^2\alpha + \beta^2\gamma + \alpha\beta\gamma) + (\gamma^2\alpha + \gamma^2\beta + \alpha\beta\gamma) - 3\alpha\beta\gamma$$

$$= \alpha(\alpha\beta + \beta\gamma + \alpha\gamma) + \beta(\beta\alpha + \beta\gamma + \alpha\gamma) + \gamma(\alpha\beta + \beta\gamma + \gamma\alpha) - 3\alpha\beta\gamma = (\alpha\beta + \beta\gamma + \gamma\alpha)(\alpha + \beta + \gamma) - 3\alpha\beta\gamma = qp - 3r$$

76.(B) α, β, γ are roots of $x^3 - px^2 + qx - r = 0$; $\alpha + \beta + \gamma = p$, $\sum \alpha\beta = q$, $\alpha\beta\gamma = r$

$$\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} \right) = \frac{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2}{\alpha^2\beta^2\gamma^2} = \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2[\alpha\beta^2\gamma + \gamma\alpha^2\beta + \alpha\beta\gamma^2]}{\alpha^2\beta^2\gamma^2} = \frac{q^2 - 2rp}{r^2}$$

77.(A) $x^2 + \left(\frac{x}{x-1} \right)^2 = 8$

$$x^2(x-1)^2 + x^2 = 8(x-1)^2$$

$$x^4 - 2x^3 + 2x^2 = 8x^2 - 16x + 8$$

$$x^4 - 2x^3 - 6x^2 + 16x - 8 = 0$$

$$(x-2)[x^3 - 6x + 4] = 0$$

$$(x-2)^2(x^2 + 2x - 2) = 0$$

Number of roots = 3

78.(B) $mx^2 - 9mx + (5m + 1) > 0 \quad \forall x \in R$

It is possible only if $m > 0$ and $D = 81m^2 - 4m(5m + 1) < 0$

$$\Rightarrow 61m^2 - 4m < 0$$

$$m(61m - 4) < 0$$

$$m \in \left(0, \frac{4}{61}\right)$$

If $m = 0$ then expression is also positive so $m \in \left[0, \frac{4}{61}\right)$

79.(B) Since, $x^4 + 20 + \sqrt{x^4 + 20} = 22 + 20$

Let $\sqrt{x^4 + 20} = y$

$$\therefore y^2 + y - 42 = 0 \quad \Rightarrow \quad y^2 + y - 42 = 0$$

$$\Rightarrow (y - 6)(y + 7) = 0 \quad \Rightarrow \quad y = 6 \quad (\because y \neq -7)$$

$$\Rightarrow \sqrt{x^4 + 20} = 6 \quad \Rightarrow \quad x^4 + 20 = 36$$

$$\Rightarrow x^4 = 16 \quad \Rightarrow \quad x = \pm 2$$

Hence, the number of real roots of the equation is 2.

80.(C) Given equation is $x^4 + x^3 - 4x^2 + x + 1 = 0$.

We should try to factorize the given equation by combining terms.

$$(x^4 - 2x^2 + 1) + (x^3 - 2x^2 + x) = 0$$

$$\Rightarrow (x^2 - 1)^2 + x(x^2 - 2x + 1) = 0 \quad \Rightarrow \quad (x - 1)^2(x + 1)^2 + x(x - 1)^2 = 0$$

$$\Rightarrow (x - 1)^2[(x + 1)^2 + x] = 0 \quad \Rightarrow \quad (x - 1)^2(x^2 + 3x + 1) = 0$$

$$\Rightarrow \text{Roots are } x = 1, 1, \frac{-3 \pm \sqrt{5}}{2} \quad \text{Positive roots are } 1, 1$$

$$\Rightarrow \text{Number of positive roots} = 2 \quad \left\{ \text{Note: } \frac{-3 - \sqrt{5}}{2}, \frac{-3 + \sqrt{5}}{2} \text{ are negative values} \right\}$$

81.(B) Let a be common root of $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$.

a will satisfy both $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$.

$$\Rightarrow a^3 + aa + 1 = 0 \quad \dots (i)$$

$$a^4 + aa^2 + 1 = 0 \quad \dots (ii)$$

Multiply (i) with a and subtract (ii) from (i) to get,

$$a - 1 = 0 \quad \Rightarrow \quad a = 1.$$

Replace $a = 1$ in (i) to get, $1 + a + 1 = 0 \Rightarrow a = -2$

Alternate Method :

By inspection, $x = 1$ satisfies both equations. $\Rightarrow x = 1$ is a common root.

Replace $x = 1$ to get $a = -2$.

82.(C) If A = first term, D = common difference of an AP, then

$$\therefore t_p = A + (p-1)D = \frac{1}{q} \quad \dots (i) \quad \text{and} \quad t_q = A + (q-1)D = \frac{1}{p} \quad \dots (ii)$$

Solving equations (i) and (ii), we get :

$$A = D = \frac{1}{pq} \quad \therefore t_{pq} = A + (pq-1)D = 1$$

Also, sum of the coefficients of the given equation is

$$(p+2q-3r) + (q+2r-3p) + (r+2p-3q) = 0$$

$$\Rightarrow x = 1 \text{ satisfies the given equation,}$$

[Using (i) and (ii)]

$$\Rightarrow x = 1 \text{ is a root of the equation.}$$

$$\text{Hence, } x = 1 = t_{pq}$$

83.(B) As α, β, γ and δ are roots of $x^4 + x^2 + 1 = 0$, we get :

$$\alpha^4 + \alpha^2 + 1 = 0 \Rightarrow (\alpha^2)^2 + (\alpha^2) + 1 = 0 \Rightarrow \alpha^2 \text{ satisfies } x^2 + x + 1 = 0 \quad \dots (i)$$

$$\beta^4 + \beta^2 + 1 = 0 \Rightarrow (\beta^2)^2 + (\beta^2) + 1 = 0 \Rightarrow \beta^2 \text{ satisfies } x^2 + x + 1 = 0 \quad \dots (ii)$$

$$\gamma^4 + \gamma^2 + 1 = 0 \Rightarrow (\gamma^2)^2 + (\gamma^2) + 1 = 0 \Rightarrow \gamma^2 \text{ satisfies } x^2 + x + 1 = 0 \quad \dots (iii)$$

$$\delta^4 + \delta^2 + 1 = 0 \Rightarrow (\delta^2)^2 + (\delta^2) + 1 = 0 \Rightarrow \delta^2 \text{ satisfies } x^2 + x + 1 = 0 \quad \dots (iv)$$

On combining (i), (ii), (iii) and (iv), we get :

$$\alpha^2, \beta^2, \gamma^2, \delta^2 \text{ are roots of } x^2 + x + 1 = 0$$

But $x^2 + x + 1 = 0$ is a degree two equation. Therefore it cannot have than two roots. It means we have to use two equations of the same nature to accommodate four roots.

$$\Rightarrow \alpha^2, \beta^2, \gamma^2, \delta^2 \text{ are roots of } (x^2 + x + 1)(x^2 + x + 1) = 0 \quad \alpha^2, \beta^2, \gamma^2, \delta^2 \text{ are roots of } (x^2 + x + 1)^2 = 0$$

84.(C) For roots to be real, Discriminant $= q^2 - 4p \geq 0 \quad \dots (i)$

Case I $p = 1$

$$\Rightarrow q^2 \geq 4 \quad \{p = 1 \text{ is replaced in (i)}\} \Rightarrow q \geq 2 \Rightarrow q = 2, 3, 4$$

$$\Rightarrow \text{order pairs } (1, 2), (1, 3), (1, 4) \text{ will make roots real} \quad \dots (ii)$$

Case II $p = 2$

$$\Rightarrow q^2 \geq 8 \Rightarrow q \geq 2\sqrt{2} \Rightarrow q = 3, 4$$

$$\Rightarrow \text{order pairs } (2, 3), (2, 4) \text{ will make roots real} \quad \dots (iii)$$

Case III $p = 3$

$$\Rightarrow q^2 \geq 12 \Rightarrow q \geq 2\sqrt{3} \Rightarrow q \geq 4 \Rightarrow q = 4$$

$$\Rightarrow \text{order pairs } (3, 4) \text{ will make roots real} \quad \dots (iv)$$

Case IV $p = 4$

$$\Rightarrow q^2 \geq 16 \Rightarrow q \geq 4 \Rightarrow q = 4$$

$$\Rightarrow \text{order pairs } (4, 4) \text{ will make roots real}$$

Combining (ii), (iii), (iv), (v) total number of order pairs that will make roots real = 7.

85.(BD) $x = 1$ is root

Let other root = α $\therefore$ Product of the roots = $(1)(\alpha) = \frac{a-b}{b-c} \Rightarrow$ roots are $1, \frac{a-b}{b-c}$

86.(A) $\pi^x = -2x^2 + 6x - 9$

$$D = 36 - 4(-2)(-9) = 36 - 72 < 0 \text{ and } a < 0$$

So quadratic expression $-2x^2 + 6x - 9$ is always negative whereas π^x is always +ve

$\therefore$ Equation will not hold for any x . $\therefore x \in \phi$

So $\pi^x = -2x^2 + 6x - 9$ has no solution.

87.(A) In the given expression, coefficient of $x^2 > 0$.

Hence, the given expression will assume positive values only if

$$[-(3a-1)]^2 - 4 \times 1 \times (2a^2 + 2a - 11) < 0 \quad [\text{Using } b^2 - 4ac < 0]$$

$$\Rightarrow 9a^2 - 6a + 1 - 8a^2 - 8a + 44 < 0 \Rightarrow a^2 - 14a + 45 < 0 \Rightarrow$$

$$(a-5)(a-9) < 0$$

$$\Rightarrow 5 < a < 9.$$

88.(B) $ax^2 + 2bx + c = 0, a_1x^2 + 2b_1x + c_1 = 0$ have a common root $\Rightarrow (ac_1 - a_1c)^2 = 4(ab_1 - a_1b)(bc_1 - b_1c)$

[From condition of one common root]

Let the discriminant of the given equation be Δ :

$$\Delta = (2bb_1 - ac_1 - a_1c)^2 - 4(b^2 - ac)(b_1^2 - a_1c_1)$$

$$= 4b^2b_1^2 + (ac_1 + a_1c)^2 - 4bb_1(ac_1 + a_1c) - 4b^2b_1^2 + 4acb_1^2 + 4a_1c_1b^2 - 4aca_1c_1$$

$$= (ac_1 + a_1c)^2 - 4aca_1c_1 - 4bb_1ac_1 - 4bb_1a_1c + 4acb_1^2 + 4a_1c_1b^2$$

$$= (ac_1 - a_1c)^2 - 4(ab_1 - a_1b)(bc_1 - b_1c) = 0 \quad \therefore \text{The roots are equal.}$$

89.(D) Since α and β are roots of $px^2 + qx + r = 0, p \neq 0$ $\therefore \alpha + \beta = \frac{-q}{p}, \alpha\beta = \frac{r}{p}$

Since, p, q and r are in AP $\therefore 2q = p + r$

$$\text{Also, } \frac{1}{\alpha} + \frac{1}{\beta} = 4 \Rightarrow \frac{\alpha + \beta}{\alpha\beta} = 4 \Rightarrow \alpha + \beta = 4\alpha\beta \Rightarrow \frac{-q}{p} = \frac{4r}{p}$$

$$\therefore 2q = p + r \Rightarrow 2(-4r) = p + r \Rightarrow p = -9r$$

$$\therefore \alpha + \beta = \frac{-q}{p} = \frac{4r}{p} = \frac{4r}{-9r} = \frac{4}{-9} \text{ and } \alpha\beta = \frac{r}{p} = \frac{r}{-9r} = \frac{1}{-9}$$

$$\therefore (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = \frac{16}{81} + \frac{4}{9} = \frac{16 + 36}{81} \Rightarrow (\alpha - \beta)^2 = \frac{52}{81} \Rightarrow |\alpha - \beta| = \frac{2}{9}\sqrt{13}$$

90.(A) Given equation are $x^2 + 2x + 3 = 0$ (i)

and $ax^2 + bx + c = 0$ (ii)

Since, equation (i) has imaginary roots. So, equation (ii) will also have both roots same as equation (i).

$$\text{Thus, } \frac{a}{1} = \frac{b}{2} = \frac{c}{3} \quad \text{Hence, } a : b : c \text{ is } 1 : 2 : 3.$$

91.(B) Let the quadratic equation be $ax^2 + bx + c = 0$

Sachin made a mistake in writing down constant terms. So, sum of roots is correct.

i.e., $\alpha + \beta = 7$

Rahul made mistake of writing down coefficient of x .

So, product of roots is correct.

i.e., $\alpha\beta = 6$

Correct quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 7x + 6 = 0$ having roots 1 and 6.

92.(A) Let α and β be the roots of equation $x^2 + ax + 1 = 0$, then $\alpha + \beta = -a$ and $\alpha\beta = 1$

Now, $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \Rightarrow |\alpha - \beta| = \sqrt{a^2 - 4}$

According to given condition, $\sqrt{a^2 - 4} < \sqrt{5} \Rightarrow a^2 - 4 < 5 \Rightarrow a^2 < 9 \Rightarrow |a| < 3 \Rightarrow a \in (-3, 3)$

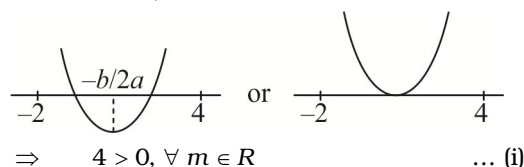
93.(AD) Use $\tan 15^\circ = 2 - \sqrt{3}$

$$\tan\left(22\frac{1}{2}\right)^\circ = \sqrt{2} - 1$$

94.(B) Since, both roots of equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 .

$\therefore D \geq 0, -2 < -\frac{b}{2a} < 4, f(4) > 0$ and $f(-2) > 0$

Now, $D \geq 0; 4m^2 - 4m^2 + 4 \geq 0$



$-2 < -\frac{b}{2a} < 4 \Rightarrow -2 < \left(\frac{2m}{2 \cdot 1}\right) < 4$

$\Rightarrow -2 < m < 4 \quad \dots (ii)$

$f(4) > 0 \Rightarrow 16 - 8m + m^2 - 1 > 0 \Rightarrow m^2 - 8m + 15 > 0 \Rightarrow (m - 3)(m - 5) > 0$

$\Rightarrow -\infty < m < 3$ and $5 < m < \infty \quad \dots (iii)$

and $f(-2) > 0 \Rightarrow 4 + 4m + m^2 - 1 > 0 \Rightarrow m^2 + 4m + 3 > 0 \Rightarrow (m + 3)(m + 1) > 0$

$\Rightarrow -\infty < m < -3$ and $-1 < m < \infty \quad \dots (iv)$

From equations (i), (ii), (iii) and (iv), we get m lie between -1 and 3

Alternate Solution :

The given equation is $x^2 - 2mx + m^2 - 1 = 0$

$\Rightarrow x^2 - 2mx + m^2 = 1 \Rightarrow (x - m)^2 = 1 \Rightarrow x - m = \pm 1 \Rightarrow x = m \pm 1$

Since, it is given that the roots of the equation are greater than -2 but less than 4 .

$\therefore -2 < x < 4 \Rightarrow -2 < m \pm 1 < 4 \Rightarrow -2 < m + 1 < 4$ and $-2 < m - 1 < 4 \Rightarrow -3 < m < 5$

and $-1 < m < 5 \Rightarrow -1 < m < 3$

95.(B) Let $f(x) = x^2 - 2kx + k^2 + k - 5$

Since, both roots are less than 5. Then, $D \geq 0$, $-\frac{b}{2a} < 5$ and $f(5) > 0$

$$\text{Now, } D = 4k^2 - 4(k^2 + k - 5) = -4k + 20 \geq 0$$

$$\Rightarrow k \leq 5 \quad \dots \text{(i)}$$

$$-\frac{b}{2a} < 5 \Rightarrow k < 5 \quad \dots \text{(ii)}$$

$$\Rightarrow 25 - 10k + k^2 + k - 5 > 0 \Rightarrow k^2 - 9k + 20 > 0 \Rightarrow (k-5)(k-4) > 0$$

$$\Rightarrow k < 4 \text{ and } k > 6 \quad \dots \text{(iii)}$$

From equations (i), (ii) and (iii), we get $k < 4$

96.(A) Since, one of the roots of equation $x^2 + px + 12 = 0$ is 4

$$\therefore 16 + 4p + 12 = 0 \Rightarrow 4p = -28 \Rightarrow p = -7$$

So, the order equation is $x^2 - 7x + q = 0$ whose roots are equal. Let the roots be α and α

$$\therefore \alpha + \alpha = \frac{7}{1} \Rightarrow \alpha = \frac{7}{2}$$

$$\text{And product of root } \alpha \cdot \alpha = q \Rightarrow \left(\frac{7}{2}\right)^2 = q \therefore q = \frac{49}{4}$$

97.(B) Let α and β be two numbers whose arithmetic mean is 9 and geometric mean is 4.

$$\therefore \alpha + \beta = 18 \text{ and } \alpha\beta = 16$$

$$\therefore \text{Required equation } x^2 - (\alpha + \beta)x + (\alpha\beta) = 0 \Rightarrow x^2 - 18x + 16 = 0$$

98.(C) Given equation $ax^2 + bx + c = 0$

Let α and β be the roots of the equation. Then, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$$\text{Also given, } \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\Rightarrow (\alpha + \beta) + \left(\frac{\alpha + \beta}{\alpha\beta}\right)^2 - \frac{2}{\alpha\beta} \Rightarrow \left(-\frac{b}{a}\right) + \left(\frac{-b/a}{c/a}\right)^2 - \frac{2}{c/a} \Rightarrow -\frac{b}{a} = \left(\frac{b}{c}\right)^2 - \frac{2a}{c}$$

$$\Rightarrow \frac{2a}{c} = \frac{b}{c} \left(\frac{b}{c} + \frac{c}{a}\right) \Rightarrow \frac{2a}{b} = \frac{b}{c} + \frac{c}{a}$$

Since, $\frac{c}{a}$, $\frac{a}{b}$ and $\frac{b}{c}$ are in AP. Hence, $\frac{a}{b}$, $\frac{b}{a}$ and $\frac{c}{b}$ are in HP.

99.(A) Since, one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, then let their roots be α and 2α .

$$\therefore \alpha + 2\alpha = -\frac{(3a-1)}{(a^2-5a+3)} \Rightarrow 3\alpha = -\frac{(3a-1)}{(a^2-5a+3)} \text{ and } \alpha \cdot 2\alpha = \frac{2}{(a^2-5a+3)}$$

$$\Rightarrow 2\alpha^2 = \frac{2}{(a^2-5a+3)} \Rightarrow \frac{(3a-1)^2}{9(a^2-5a+3)} \Rightarrow (3a-1)^2 = 9(a^2-5a+3)$$

$$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27 \Rightarrow 45a - 6a = 27 - 1 \Rightarrow a = \frac{26}{39} = \frac{2}{3}$$

100.(A) Given that $p(x) = f(x) - g(x)$ has only one root -1 .

$$\Rightarrow p(x) = (a - a_1)x^2 + (b - b_1)x + (c - c_1) \text{ has one root, } -1 \text{ only,}$$

$$\Rightarrow p'(x) \text{ will also have root as } -1 \Rightarrow p'(x) = 0 \text{ at } x = -1$$

$$\Rightarrow 2(a - a_1)x + (b - b_1) = 0 \text{ at } x = -1 \Rightarrow -2(a - a_1) + (b - b_1) = 0 \Rightarrow \frac{-(b - b_1)}{(a - a_1)} = -2 \quad \dots (i)$$

$$\text{Now, } p(x) = (a - a_1)x^2 + (b - b_1)x + (c - c_1)$$

$$\Rightarrow \frac{p(x)}{a - a_1} = x^2 + \frac{b - b_1}{a - a_1}x + \frac{c - c_1}{a - a_1} \quad \therefore p(-1) = 0$$

$$\therefore 0 = (-1)^2 + \frac{b - b_1}{a - a_1} + \frac{c - c_1}{a - a_1} \Rightarrow 0 = 1 - 2 + \frac{c - c_1}{a - a_1} \quad [\text{Using equation (i)}]$$

$$\Rightarrow \frac{c - c_1}{a - a_1} = 1 \quad \dots (ii)$$

$$\text{Also } p(-2) = 2$$

$$\Rightarrow 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2 \quad \dots (iii)$$

From equations (i), (ii) and (iii), we have

$$4(a - a_1) - 4(a - a_1) + (a - a_1) = 2$$

$$(a - a_1) = 2$$

On substituting $a - a_1 = 2$ in equations (i) and (ii), we get :

$$c - c_1 = 2, \quad b - b_1 = 4$$

$$\text{Now, } p(2) = 4(a - a_1) + 2(b - b_1) + (c - c_1) = 8 + 8 + 2 = 18$$

101.(A) $ax^2 + bx + 1 = 0$ roots $\frac{1}{\sqrt{\alpha}}, \frac{1}{\sqrt{\beta}}$

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = -\frac{b}{a}, \quad \frac{1}{\sqrt{\alpha\beta}} = \frac{1}{a}$$

$$a = \sqrt{\alpha\beta}$$

$$b = -(\sqrt{\alpha} + \sqrt{\beta})$$

$$x^2 + [-(\sqrt{\alpha} + \sqrt{\beta})^2 + 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta})] + \alpha^{3/2} \cdot \beta^{3/2} = 0$$

$$(x - \alpha^{3/2})(x - \beta^{3/2}) = 0$$

$$x = \alpha^{3/2}, x = \beta^{3/2}$$

102.(A) $\sqrt{2x+1} - \sqrt{2x-1} = 1$ rationalizing

$$\Rightarrow \frac{2}{\sqrt{2x+1} + \sqrt{2x-1}} = 1 \Rightarrow \sqrt{2x+1} + \sqrt{2x-1} = 2$$

$$\Rightarrow \sqrt{2x+1} = \frac{3}{2}, \quad \sqrt{2x-1} = \frac{1}{2} \Rightarrow \sqrt{4x^2 - 1} = \frac{3}{4}$$

103.(B) The equation $(x^2 - 2x)^2 - 3(x^2 - 2x) + k + 2 = 0$, has four real and distinct roots

if $t^2 - 3t(k + 2) = 0$... (i) and 't' must lie in $(-1, \infty)$ where $t = x^2 - 2x \geq -1$

Let the roots of (i) be α and β .

And for t to lie in $(-1, \infty)$, α, β both lie greater than -1 (i.e. both roots α, β greater than -1)

So, $a.f(-1) > 0, D > 0, \frac{-b}{a} > -1$

Now, $1.f(-1) > 0 \Rightarrow 1 + 3 + k + 2 > 0 \Rightarrow k > -6$... (ii)

And $D > 0 \Rightarrow D = 9 - 4(k + 2) > 0 \Rightarrow k < \frac{1}{4}$... (iii)

Also, $\frac{-b}{a} > -1$, Which is always true. ... (iv)

Now from (ii), (iii) and (iv) we get : $k \in \left(-6, \frac{1}{4}\right)$